Saving the US from a Missile Crisis.

I. ASSESSMENT TASK OVERVIEW & PURPOSE:

In this activity, students will be asked to write a quadratic equation given two points and the vertex. They will then have to create a quadratic equation that will intersect the parabola above a certain height. Students will discover multiple possibilities and then solve the system of equations. They must also draw a graph representing their system. At the end of the task, students will present their posters to the rest of the class.

II. UNIT AUTHOR:

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III. COURSE:

Algebra II

IV. CONTENT STRAND:

Equations and Inequalities

V. OBJECTIVES:

The student will be able to:

- Write quadratic equations
- Solve the system of equations
- Graphically represent the situation

VI. REFERENCE/RESOURCE MATERIALS:

Calculator, Graph paper, Poster Board

VII. PRIMARY ASSESSMENT STRATEGIES:

The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. The assessment list for this activity is intended to evaluate the structure of the quadratic equations, and the graphical representation. It will also evaluate the students' group work and final class presentation.

VIII. EVALUATION CRITERIA:

Assessment List for Activity 1, corresponding rubric.

IX. INSTRUCTIONAL TIME:

This Activity is estimated to take 2-90 minute blocks or 4-45 minute classes.

Saving the US from a Missile Crisis

Strand

Equations and Inequalities

Mathematical Objective(s)

This activity will address the concept of writing quadratic equations. Students will solve a system of equations. Students will also develop skills necessary to identify and evaluate different solutions to systems of equations. Students will also develop skills necessary to represent and analyze real-world situations mathematically and graphically.

Related SOL

• AII.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

NCTM Standards

- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases;
- Draw reasonable conclusions about a situation being modeled.

Materials/Resources

- Graphing Calculator
- Graph Paper
- Poster Board
- Background/Instructions Page

Assumption of Prior Knowledge

- Students should have basic understanding of the formula to write a quadratic equation given roots and vertex.
- Students should have basic understanding of how to graph quadratic equations.
- Students should have basic understanding of paths for projectiles.
- Students should feel comfortable working in groups and drawing graphs to represent the situation.
- Students should feel comfortable speaking in front of the entire class.
- Students may have difficulty writing a quadratic equation that will intersect the projectile above a specific height.
- Students may have difficulty drawing a graph to represent the real-world situation.
- Students may have difficulty or feel nervous about speaking in front of the class.
- The relevant context students should have already explored is the Cuban Missile Crisis.

Introduction: Setting Up the Mathematical Task

• In the task, you will investigate real-world situations of solving a system of equations. To begin, you will be given an instructional page that sets the background for this task, which will discuss the Cuban Missile Crisis. In order to save the U.S. you must create a quadratic equation that will represent the projectile path that will destroy the bomb above a safe height. In your groups, you will discuss the necessary actions that need to be taken, such as drawing the missile projectile and finding a quadratic equation that will intersect the path. After gathering all the necessary information, your group will draw a

picture and graph of the real-world situation on poster board. At the end of the task, your group will present the poster to the class and discuss how your group came to that conclusion. You will be given roughly two days to gather information and draw a graphical representation. Then you will have roughly two days to present your finding to the class.

- Teacher will invite students to draw upon prior knowledge of the Cuban Missile Crisis by allowing a brief discussion of the topic. Also, a warm-up activity will be completed to help students understand the formula for writing quadratic equations.
- Teacher will guide students during the task by asking questions such as "What is the shape of the projectile path?" "What do you need to write a quadratic equation?" "What is the formula to write a quadratic equation given roots and a vertex?"
- Students will be working in groups. Within their groups, they will brainstorm different ideas to develop a system of equations. They will use cooperative learning and assign duties to each person in the group.
- Writing equations to develop a system of equations and representing the findings graphically help students develop necessary skills to be successful on the Algebra II SOL.
- Presenting the posters of their group's findings will allow students to make their mathematical thinking and understanding public.

Student Exploration

Individual Work

- Students will complete a Warm-Up activity individually that activates prior knowledge of writing quadratic equations.
- Students will also be assigned different duties within groups that they are responsible for individually.

Small Group Work

- Students will work in small groups to accomplish the task of representing a real-world situation graphically.
- Students will gather information needed to write a system of equations.
- Students will then solve the system of equations.
- Finally, students will draw a graph on poster board to represent the situation and findings.

Whole Class Sharing/Discussion

- Students will present their posters and findings to the entire class.
- Students will question each other's finding to gain a better understanding of different possibilities for finding a solution.

Student/Teacher Actions:

- Students will be working collaboratively in groups to find a solution to a system of equations.
- The teacher should be monitoring progress and ensure students are working collaboratively. In addition, the teacher should ask questions to confirm students' understanding.
- One common misconception the students may experience is the intersection of the two projectile paths must be above a certain height. They might not understand why the intersection of the two equations has to be above a certain height, or they might not understand how to ensure the intersection of the two equations is above that height.
- In order to address the misconception, the teacher should explain the consequences of blowing a bomb up too close to the ground.
- One way to increase student learning is to incorporate the use of computers for research on the topic. Also, the use of graphing calculators should be promoted.

Monitoring Student Responses

- Students are to communicate their thinking and their new knowledge by drawing and presenting posters to the class
- Students are to communicate with each other by working cooperatively in groups.
- Teacher and/or students are to highlight and clarify the ideas being grappled by answering questions asked by students.
- Teacher is to assist students who have difficulties by providing guidance and further explanations.
- Teacher is to extend the material for students that are ready to move forward and emphasize realworld application.

Summarize of the task/activity

- At the end of the task, students will be asked to present their findings to the class. Once all groups have presented, students will be asked to summarize all findings on an exit slip.
- In order to collect evidence of students' knowledge of the content described, the group posters created by the students will be displayed on the classroom walls. These posters will display the two equations for the projectiles, two graphs, the intersection point of the two projectiles, and other helpful information.

Assessment List and Benchmarks

Warm-up Activity:

- 1. Write a quadratic equation given the points (2,4) (3,9) (10,100). 3pts.
- 2. Write a quadratic equation given the roots 15 and 30 and also the vertex (24, 34). 3pts.
- 3. Solve this system of equations: $\begin{cases} x^2 + 3x 9 \\ -4x^2 + 5x + 6 \end{cases}$ 2pts.
- 4. Solve this system of equations: $\begin{cases} 2x^2 + x 10 \\ -10x^2 + 2x + 8 \end{cases}$ 2pts.
- 5. Draw a rough graph of the projectile path of a ball being shot out of a canon. 5pts.

Task 1: Solving System of Equations

In your groups, each person will be assigned a job. There are listed below:

- Manager- oversees the entire project to ensure accuracy.
- **Coach** ensures everyone agrees on an answer.
- **Book keeper** responsible of writing what everyone has agreed on.
- Editor- ensures there are no errors, grammatically, graphically, or mathematically.
- **Referee** ensures everyone is working together peacefully.

Each person in the group must participate and speak in the final presentation. In order to gain full create, all categories on the rubric must be completed with accuracy. Use the rubric to self-assess to ensure all categories are met with full potential. Highlight the appropriate score you think your group should earn. Be sure to grade yourselves with honesty! This will be a group grade so collaboration is highly encouraged!

At the end of the task, your group must create a poster drawing of the real-world situation. In addition, your group must present these findings to the class. You must describe how you arrived at both equations. In addition, each

person in the group must explain some part of the task. Be sure to discuss the point of intersection on your two graphs and also the solution to your system. Compare and contrast these two points.

Saving the US from a Missile Crisis

<u>Directions:</u> Today you are fighting to save the United States. Cuba has just announced they are going to fire a missile aimed at Washington D.C. In order to save the U.S., you and your group must fire a shot to take the missile out. However, be careful not the hit the missile too low to the ground. If you hit the missile below 12,000 feet, this will cause anything on the ground within a 100 mile radius to be destroyed. Be strategic in taking the missile out and know there are multiple possibilities for the path for your shot.

- 1. Cuba and Washington DC are approximately 1210 miles apart. When the missile is fired, its vertex of its path is estimated to be halfway between Cuba and Washington at an elevation of 45,000 feet. Using the locations of Cuba and Washington DC as roots for your equation, write an equation to represent the projectile path from Cuba to Washington D.C. Graph your equation and make a small scale model on a piece of graph paper.
- 2. In order to save the U.S. a shot must be fired to take the missile out. This shot must be fired with accuracy and hit the missile at an elevation above 12,000 feet. This will ensure no innocent bystanders are injured upon explosion. Write an equation that will represent the path of the shot being fired at the missile. Graph this equation and copy it onto your graph paper. Label the point(s) of intersection.
- 3. Using the equations that you created, solve the system of equations. Once solved, label this point on your graph.
- 4. Compare the original point of intersection with the solution to the system. Are they the same? If not, why do you think they are different?
- 5. What were the original limitations of the problem? How was the problem resolved by using these limitations? What are some other ways in which the problem might have been approached?
- 6. After checking all work, draw a completed illustrated representation of the situation. Be sure to draw both graphs. Label the point of intersection of the two graphs. Also, label the solution of the system of equations. Be creative and make the posters eye appealing. Include geographic pictures to represent the US and Cuba. You can also use a computer to obtain pictures for your posters.

Rubric:

RUBRIC FOR TASK 1											
Gro	up Members:	Assessment Grade									
#	Element	0	1	2	Self	Teacher					
	Title and description	No title or	Title and	Title and							
1		description provided	description are	description							
			incomplete	provided							
	1 st Quadratic Equation	Quadratic equation	Quadratic equation	Quadratic							
2		not stated	stated but incorrect	equation stated							
				and accurate							
	2 nd Quadratic Equation	Quadratic Equation	Quadratic equation	Quadratic							
3		not stated	stated but incorrect	equation stated							
				and accurate							

TOTAL POINTS EARNED						/20
10	Presentation of Findings	No presentation	Presented findings but not accurately	Presented findings accurately		
9	Speech	No Speech	Not everyone spoke clearly and understandably	Everyone spoke clearly and was understandable		
8	Group Duty	Did not complete duty	Completed duty but accurately	Completed duty accurately		
7	The components of the project are organized.	No organization	Some organization	Materials are well- organized when handed in.		
6	Neatness of materials	Materials are not neat	Materials are lacking in neatness	Materials are neat and easy to read.		
5	Graph of 2 nd quadratic	No Graph of quadratic	Graph of quadratic drawn but not correct	Graph of quadratic drawn and correct		
4	Graph of 1 st quadratic	No Graph of quadratic	Graph of quadratic drawn but not correct	Graph of quadratic drawn and correct		

Exit Slip:

- 1. After reviewing every group's findings, was there a common location of intersection?
- 2. Did any group have the same system of equations? If so, was their solution the same?
- 3. Summarize the general findings of each group's presentation.

Sample Student Work:

Saving the US from a Missile Crisis

<u>Directions:</u> Today you are fighting to save the United States. Cuba has just announced they are going to fire a missile aimed at Washington D.C. In order to save the U.S., you and your group must fire a shot to take the missile out. However, be careful not the hit the missile too low to the ground. If you hit the missile below 12,000 feet, this will cause anything on the ground within a 100 mile radius to be destroyed. Be strategic in taking the missile out and know there are multiple possibilities for the path for your shot.

1. Cuba and Washington DC are approximately 1210 miles apart. When the missile is fired, its vertex of its path is estimated to be halfway between Cuba and Washington at an elevation of 45,000 feet. Using the locations of Cuba and Washington DC as roots for your equation, write an equation to represent the projectile path from Cuba to Washington D.C. Graph your equation and make a small scale model on a piece of graph paper.

Roots would be (0, 0) and (0, 1210). The vertex would be (605, 45000) since the midpoint of 0 and 1210 is 605. Using $y=a(x-h)^2+k$, this means the equation for the parabola would be

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0=a (0-605)<sup>2</sup> +45000
0=366025a+45000
-45000=366025a
-0.123=a
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This means my quadratic equation would be: $y = -0.123(x-605)^2+45000$

2. In order to save the U.S. a shot must be fired to take the missile out. This shot must be fired with accuracy and hit the missile at an elevation above 12,000 feet. This will ensure no innocent bystanders are injured upon explosion. Write an equation that will represent the path of the shot being fired at the missile. Graph this equation and copy it onto your graph paper. Label the point(s) of intersection.

To shoot the missile we are going to hit it before it reaches its vertex to ensure we will have enough time to shoot again if we miss. Also, we will hit the missile above a height of 12000 to help minimize the risk of injuring bystanders on the ground. To begin, we located a point on the graph above a height of 24000, so we chose x=1000. Plugging in:

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y=-0.123(1000-605)^2+45000

y=-0.123(395)^2+45000

y=-0.123(156025)+45000

y=-19191.075+45000

y=25808.925\approx 25809
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This means my point of intersection is going to be (1000,25809). I also need one more point in order to write my equation. I am going to choose (600,30000). Now that I have my three points I can use the calculator to find my equation. I found my equation to be $y=-0.0605x^2+86.2864x$

3. Using the equations you created, solve the system of equations. Once solved, label this point on your graph.

Graphically I got (1000,25809) and (0.337,29.1)

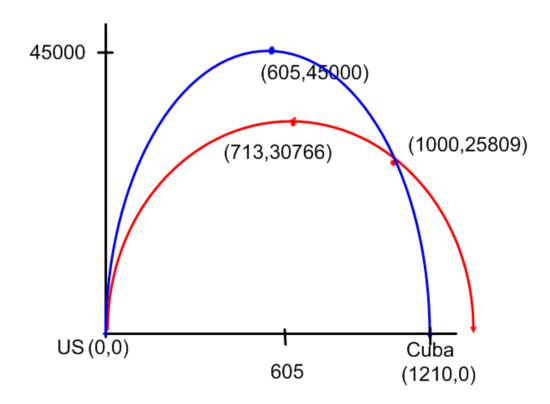
4. Compare the original point of intersection with the solution to the system. Are they the same? If not, why do you think they are different?

The solution is the same for both.

5. What were the original limitations of the problem? How was the problem resolved by using these limitations? What are some other ways in which the problem might have been approached?

To begin, the equations must intersect at a height above 12,000 feet. Also, we know they had to be quadratic from our prior knowledge of projectiles. This allowed us to use one point of the graph to write our equation. The problem could have been solved using a graphing calculator and scatterplot.

6. After checking all work, draw a completed illustrated representation of the situation. Be sure to draw both graphs. Label the point of intersection of the two graphs. Also, label the solution of the system of equations. Be creative and make the posters eye appealing. Include geographic pictures to represent the US and Cuba. You can also use a computer to obtain pictures for your posters.



Exit Slip:

1. After reviewing every group's findings, was there a common location of intersection?

There was not a common location of intersection because we all used different equations. Some groups intersected the missile at a higher or lower height which changed the location.

- 2. Did any group have the same system of equations? If so, was their solution the same? No group had the same system of equations. However, some were very similar, with points of intersections very close to one another.
- 3. Summarize the general findings of each group's presentation. It was determined that there are numerous possibilities to intersect the missile to ensure safety. All of these intersection points were located above a height of 12,000, but with a range of about (100-1100).



FACT SHEET

U.S. Air Force Fact Sheet CUBAN MISSILE CRISIS

The Cuban Missile Crisis of 1962 dramatically proved the importance of the <u>U-2</u> and aerial reconnaissance. On Oct. 14, 1962, two USAF U-2s photographed portions of Cuba, revealing Soviet offensive nuclear missiles based only 90 miles from U.S. shores. President John F. Kennedy placed U.S. forces on alert, and USAF U-2 and RF-101 reconnaissance flights over Cuba continued, the latter aircraft sometimes flying at treetop level. On Oct. 22, President Kennedy publicly announced details of the critical situation and ordered a naval blockade of Cuba.

Meanwhile, USAF aircraft kept the island and surrounding waters under constant surveillance, providing the U.S. Navy with data on scores of ships at sea apparently en route to Cuba. On Oct. 27, USAF Maj. Rudolf Anderson Jr. was shot down and killed flying a U-2 mission over Cuba. The superpowers were then very close to war. The next day, Soviet Premier Nikita Khrushchev -- faced with U.S. resolve to prevent Soviet strategic weapons being placed so close to the United States -- agreed to remove the offensive missiles as well as medium range bombers being assembled in Cuba. USAF U-2s and RF-101s then monitored communist compliance in removing this threat to American security.

Click here to return to the U-2 Overview.



Maj. Rudolf Anderson Jr. was shot down and killed over Cuba during the October 1962 crisis. He was flying a U-2 from McCoy AFB, Fla., and was brought down by a Soviet SA-2 missile. Anderson was posthumously awarded the first Air Force Cross, which had been created in 1960. Anderson and other Strategic Air Command and Tactical Air Command pilots provided pictures that gave U.S. leaders crucial information and proved to the world that offensive nuclear missiles were being placed in Cuba. (U.S. Air Force photo)

^{*}Obtained from http://www.nationalmuseum.af.mil/factsheets/factsheet.asp?id=13888